Models of Computation: Automata and Processes

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Motivation – "Beyond Turing"

- Automata theory: simple models of computation
  - Understanding the *principles* of computing
  - Analysis of *computability*, complexity
- Process theory: origins in automata theory
  - “No interaction with environment”
  - Focus: notion of *interaction* and parallel behaviour

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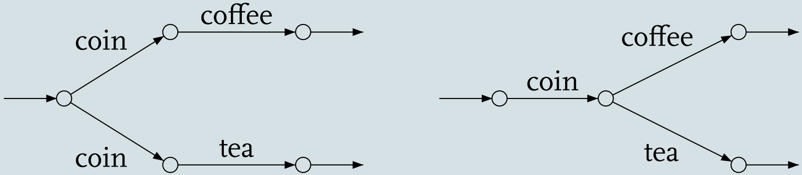
1. Goal: *integration* of automata and process theory
   - Attempt reveals differences and similarities
   - Use *analogies* to make the integration explicit
2. Goal: Add process theory to the undergraduate curriculum

Automata

Automata accept languages as correct or wanted behaviour:



The above automata accept the same language, they are *language equivalent*:

- a **coin** followed by **coffee**
- a **coin** followed by **tea**

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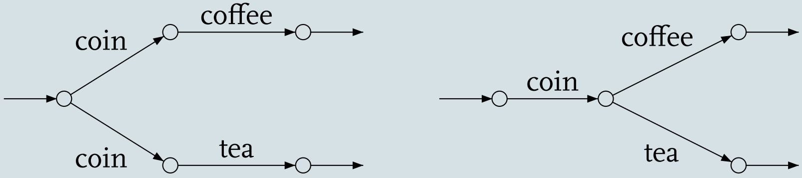

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Process theory differentiates using the *bisimulation equivalence*

Regular Expressions and Process Terms

- Regular expressions describe languages:

$$\text{coin} \cdot \text{coffee} + \text{coin} \cdot \text{tea}, \qquad \text{coin} \cdot (\text{coffee} + \text{tea})$$

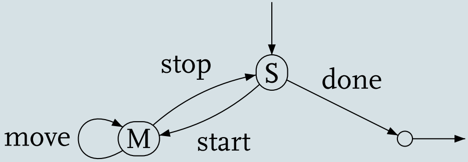
- Regular expressions can describe all regular languages
- Their process term counterparts cannot!
- Process terms have calculation rules (axioms). E.g.:

$$\begin{array}{lrl} (A3) & x + x & = x \\ (A4) & (x+y)z & = xz + yz \end{array}$$

- Process theory: additional operators ($\|$, $|$, and $\lfloor\!\lfloor$) for describing parallel behaviour which are *not* present in automata theory.

Grammars and Recursive Specifications

The context-free process S:



- Grammars can also describe formal languages
- Right-linear grammars are equivalent to recursive specifications

Grammars and Recursive Specifications

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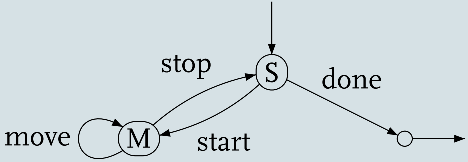

We can give both for the automaton above:

$$\begin{array}{ll|ll} S & \rightarrow start\ M\ S \mid done & S & = start \cdot M \cdot S + done \\ M & \rightarrow move\ M \mid stop & M & = move \cdot M + stop \end{array}$$

Preliminary Result

(Jos Baeten, Bas Luttik, Clemens Grabmayer)

$$\begin{array}{ll|ll} S & \rightarrow start\ M\ S \mid done & S & = start \cdot M \cdot S + done \\ M & \rightarrow move\ M \mid stop & M & = move \cdot M + stop \end{array}$$

- Automata theory: context-free language can be accepted by *push-down automaton*
- This specialised automaton employs a stack
- Process theory: context-free process can be transformed into a regular process communicating with a Stack process

$$\begin{aligned} S &= start.push(S).M + done.E_\theta \\ M &= move.M + stop.E_\theta \\ E_\theta &= pop(V).V + empty \end{aligned}$$

Research Questions

- New operators, new languages: expressiveness of these new languages?
- Finite axiomatisations?
- Extension of Chomsky hierarchy?
- More transformations?

Research Team

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Questions?