Formal Languages, Automata and Processes

Finished and Future Work

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(joint work with Jos Baeten, Bas Luttik and Pieter Cuijpers)

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Department of Mathematics and Computer Science
Eindhoven University of Technology

IPA Fall Days, 2008

Project MoCAP

- Models of Computation: Automata and Processes

  Automata + Interaction = Concurrency

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  Automata + Interaction = Concurrency

- Separate development
- Integration
  - Attempt reveals differences and similarities
  - Use *analogies* to make the integration explicit

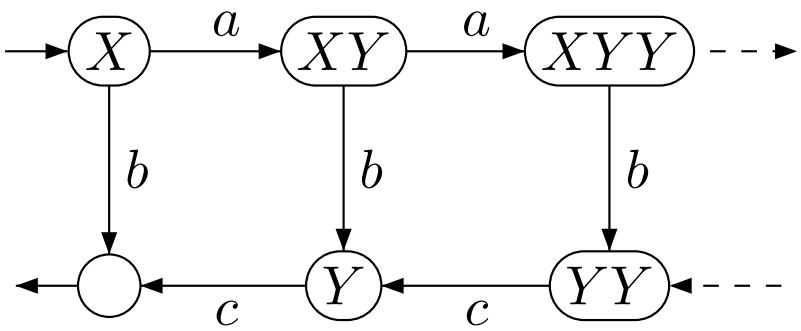
Example: Context-Free Language

Right-linear grammar

Generates a context-free language

$$X \longrightarrow aXY \mid b$$
$$Y \longrightarrow c$$

Transition system



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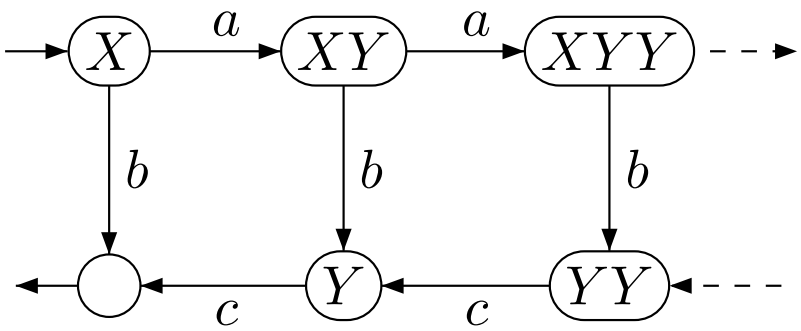

Theorem

For every context-free language there exists a pushdown automaton that accepts it.

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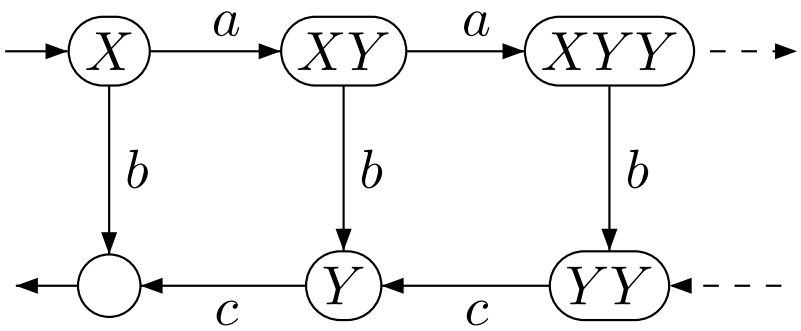
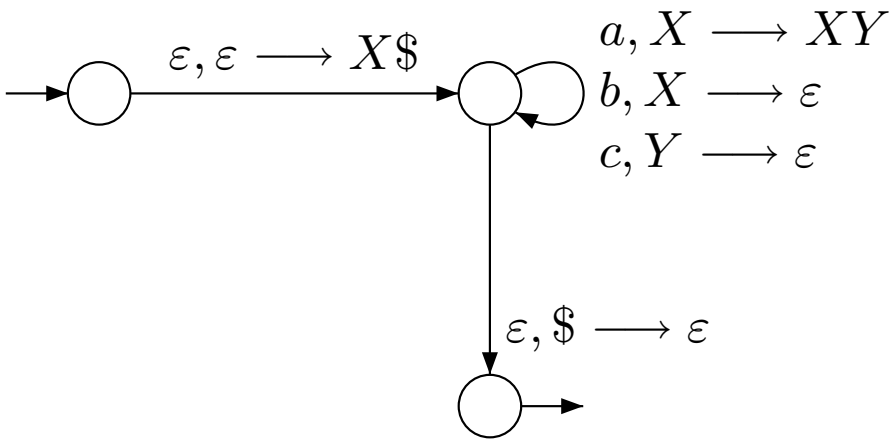
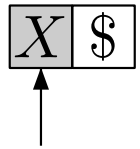


Pushdown automaton



Stack



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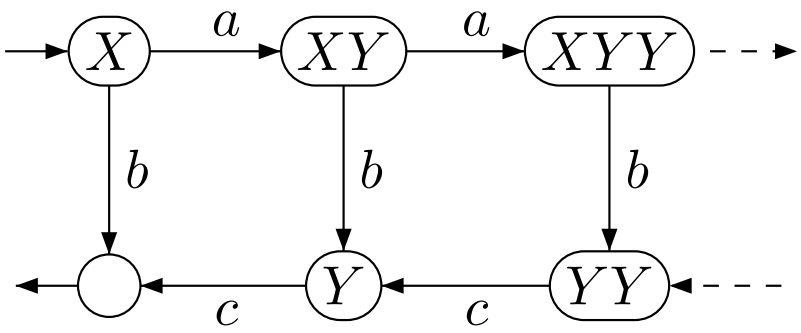

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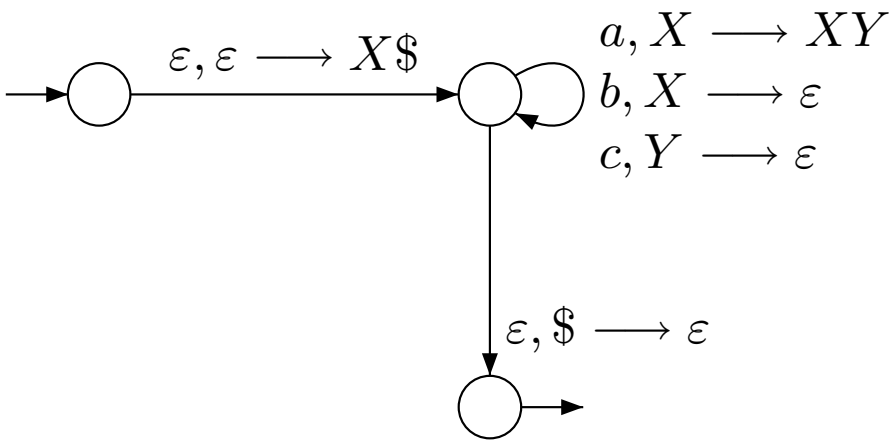

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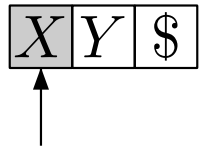

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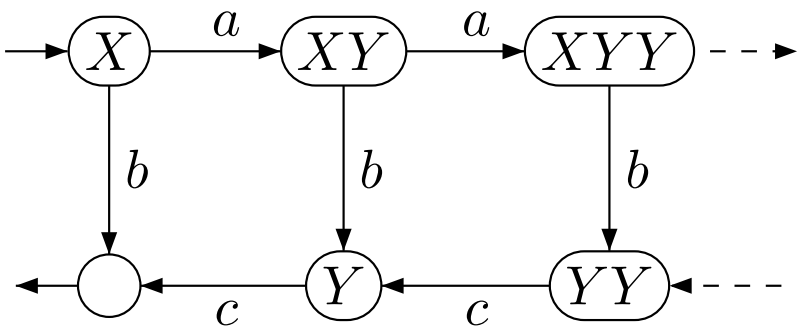

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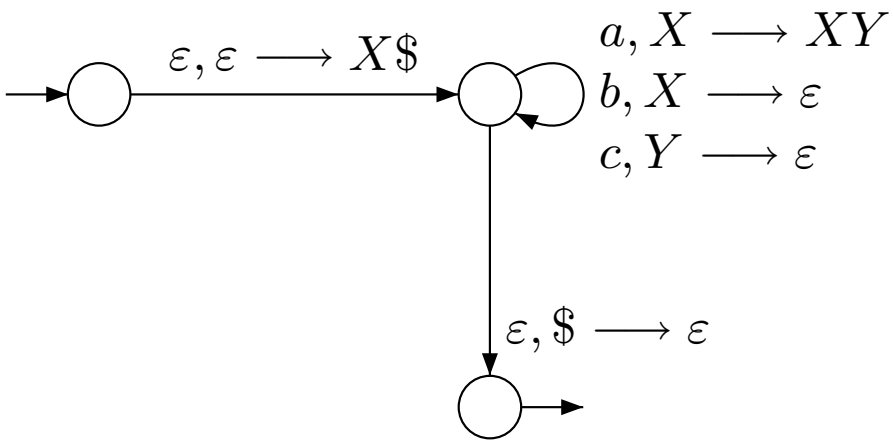

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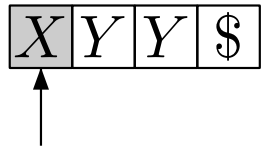

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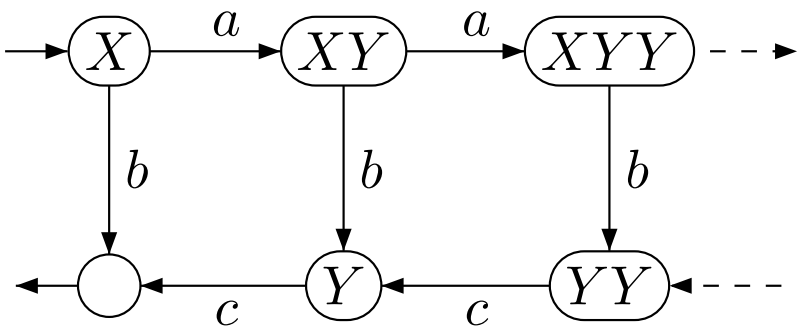

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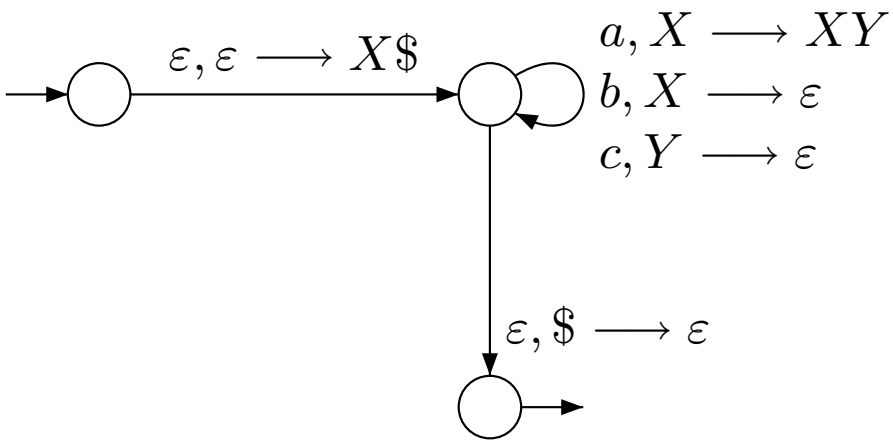

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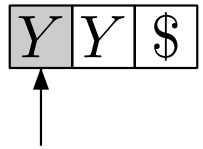

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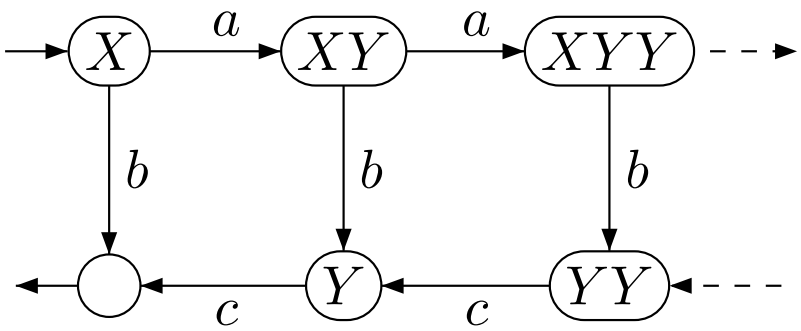

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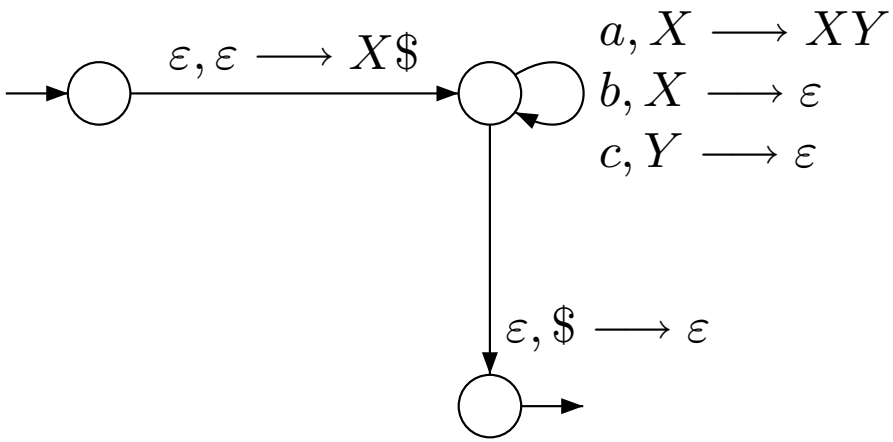

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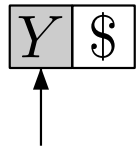

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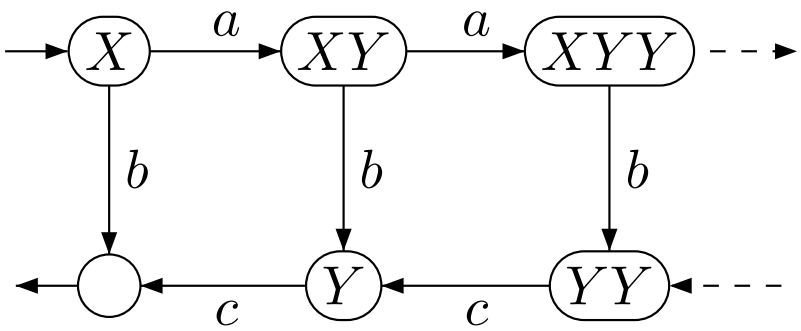

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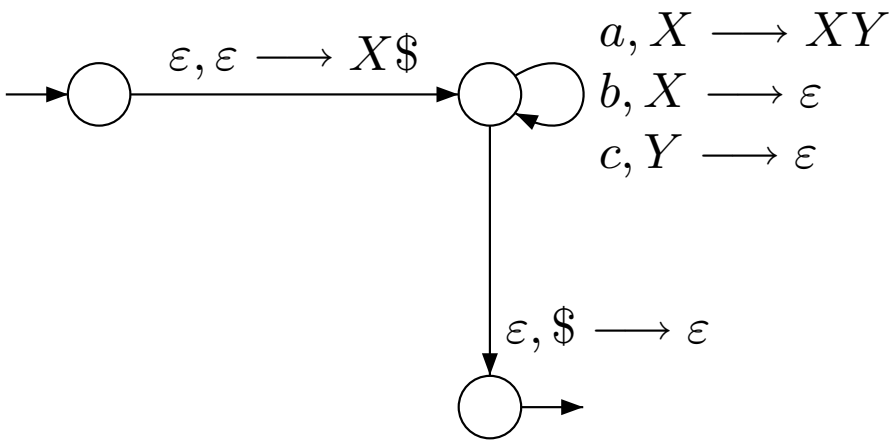

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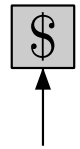

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Recursive specification over BPA

Specifies a context-free process

$$X = a \cdot (X \cdot Y) + b$$
$$Y = c$$

Restrict to:
finite and guarded specifications

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- Regular expressions use 0 (deadlock) and 1 (final state)
- Not done in grammars in automata theory
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Processes and Recursive Specifications

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Main Theorem

Process theory enables us to...

- Model the data (a stack) as a process
- Make communication explicit
- Use bisimulation equivalences to preserve branching structure

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- Model the data (a stack) as a process
- Make communication explicit
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Theorem

Every context-free process is equivalent to a regular process communicating with a stack. [CONCUR08]

Pushdown Automaton

Context-free process

$$X = a.(X \cdot Y) + b.\mathbf{1}$$
$$Y = c.\mathbf{1}$$

Translated

$$\hat{X} = a.\mathrm{Push}(XY) + b.\mathrm{Push}(\mathbf{1})$$
$$\hat{Y} = c.\mathrm{Push}(\mathbf{1})$$

$$\mathrm{Push}(\mathbf{1}) = \mathrm{Ctrl}$$
$$\mathrm{Push}(\xi Y) = !Y.\mathrm{Push}(\xi)$$
$$\mathrm{Ctrl} = \sum_{V \in \mathcal{V}} ?V.\hat{V} + \mathbf{1}$$
$$S = \mathbf{1} + \sum_{V \in \mathcal{V}} ?V.S \cdot !V.S$$

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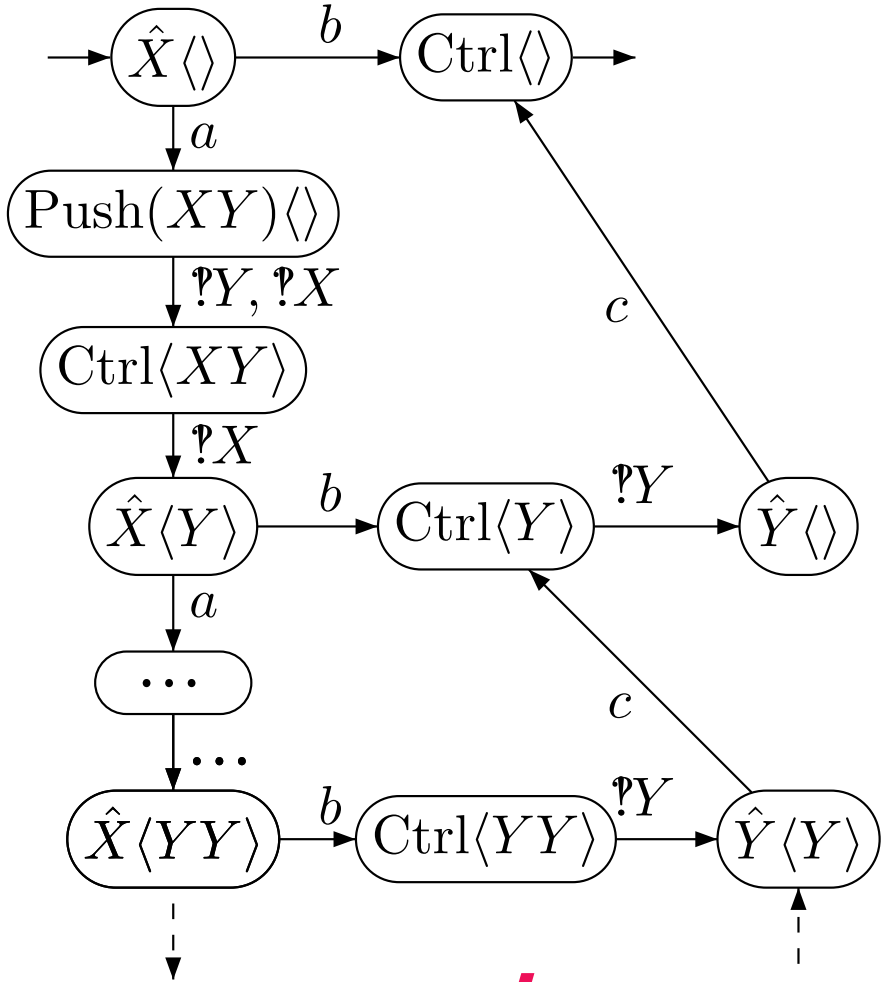

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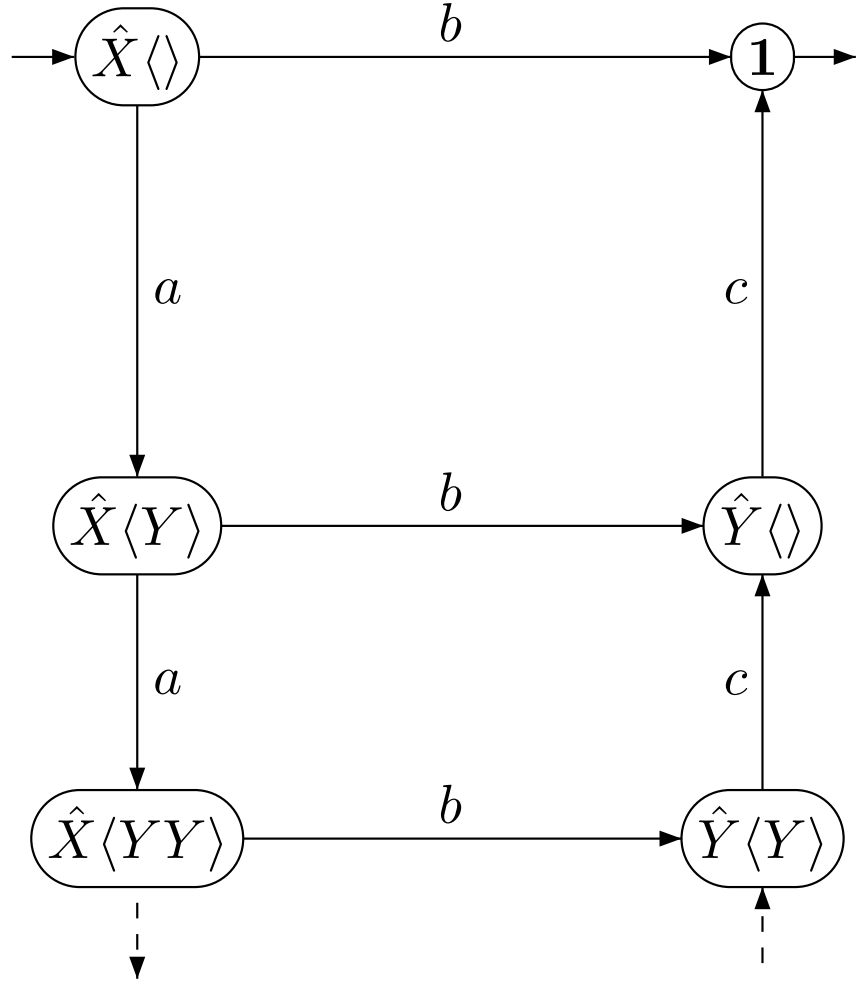
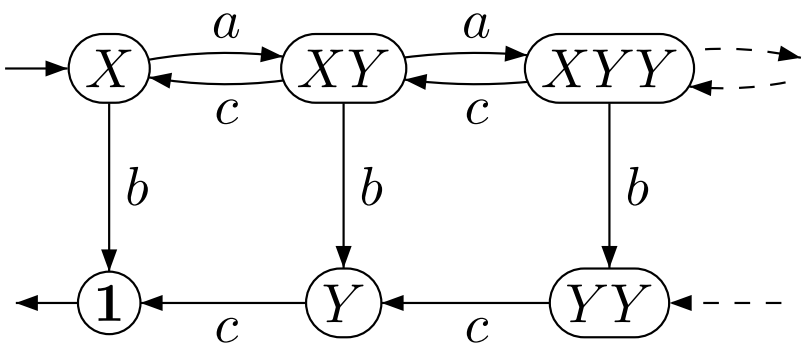
... modulo rooted br. bisim.



Recursive specification over BPP

$$X = a.(X \parallel Y) + b.\mathbf{1}$$
$$Y = c.\mathbf{1}$$

Transition system

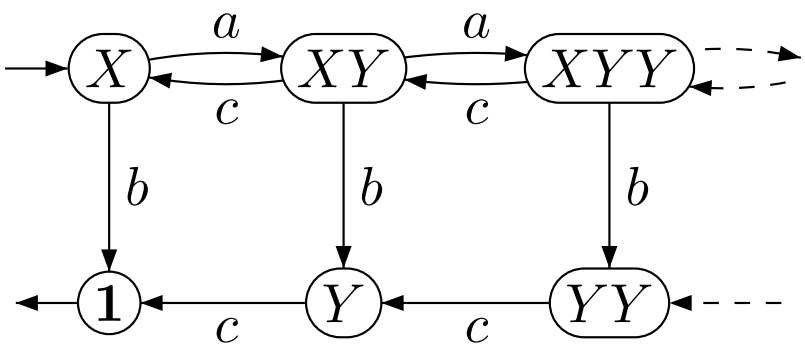


Basic Parallel Processes

Recursive specification over BPP

$$X = a.(X \parallel Y) + b.\mathbf{1}$$
$$Y = c.\mathbf{1}$$

Transition system



Theorem

Every basic parallel process is equivalent to a regular process communicating with a bag. [EXPRESS08]

Future work

Basic Queueing Processes

- Future work with master students
- Prove a similar theorem

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Expressivity

- Relative expressivity of BPA_0, BPP_0 and PA_0 known
- Complete the picture with 1

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- Prove a similar theorem

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Decidability

- Decidability of bisimulation for BPA known
- Investigate with 0 and 1

Thank you!

Questions?